

REPRESENTATIVE DYNAMICS

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This short note is devoted to the representative dynamics, which realizes a link between the theory of controlled systems and representation theory. Dynamical inverse problem of representation theory for controlled systems is considered: to solve it means to correspond a representative dynamics to the controlled system.

This short note is devoted to the representative dynamics, a new concept, which realizes a link between the theory of controlled systems and representation theory, and the dynamical inverse problem of representation theory for controlled systems. Thus, it may be considered as a development of ideas of the earlier article [1], which follows the general ideology of inverse problems of representation theory [2].

1. Representative dynamics.

Definition 1. Let $\mathbf{X} = \mathbf{X}(t) = (X_1(t), \dots, X_m(t))$ ($X_i(t) \in \text{Mat}_n(\mathbb{C})$) be the time-dependent vector of m complex $n \times n$ matrices. The *representative dynamics* is a controlled system (with constraints) of the form

$$(1) \quad \dot{\mathbf{X}}(t) = F(\mathbf{X}(t), a(t))$$

with the *fixed* initial data $\mathbf{X}(t_0)$, where the control parameter $a(t) = (\mathfrak{A}(t), \mathbf{e}(t))$ is the pair of *any* associative algebra $\mathfrak{A}(t)$ from the fixed class of such algebras \mathbb{A} , $\mathbf{e}(t) = (e_1(t), \dots, e_m(t))$ is *any* set of algebraic generators of the algebra \mathfrak{A} (one may claim $\mathbf{e}(t)$ to be an algebraic basis in \mathfrak{A}) such that the mapping $e_i(t) \mapsto X_i(t)$ may be extended to the representation $T(t) : \mathfrak{A}(t) \mapsto \text{Mat}_n(\mathbb{C})$ of the algebra $\mathfrak{A}(t)$ in the matrix algebra $\text{Mat}_n(\mathbb{C})$ (this is a constraint on the control $a(t)$).

Certainly, the claim that (1) is a representative dynamics restricts the choice of the function F and initial data $\mathbf{X}(t_0)$ because for each moment t any admissible choice of the pair $(\mathfrak{A}(t), \mathbf{e}(t))$ should provide that the set of admissible pairs will not be empty in future.

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Exercise. To describe all representation dynamics with $m=3$, $n=2$ and the class \mathbb{A} of all associative algebras \mathfrak{A} with quadratic relations, which are isomorphic as linear spaces to the symmetric algebra $S(V)$ over the linear space V spanned by the fixed elements e_i of \mathfrak{A} under the Weyl symmetrization mapping.

Remark 1. Let us consider the following equivalence on the set \mathcal{A} of all admissible $a = (\mathfrak{A}, \mathbf{e})$. The pairs $a_1 = (\mathfrak{A}_1, \mathbf{e}_1)$ and $a_2 = (\mathfrak{A}_2, \mathbf{e}_2)$ will be equivalent iff the algebras A_1 and A_2 are isomorphic under an isomorphism which maps the linear space V_1 spanned by the elements of \mathbf{e}_1 onto the linear space V_2 spanned by the elements of \mathbf{e}_2 . Then the equivalence divides the time interval $[t_0, t_1]$, on which the representative dynamics is considered, onto the subsets, on which the pairs $a(t)$ are equivalent.

Remark 2. Representative dynamics combines structural and functional features. The first are accumulated in the class \mathbb{A} and the least are expressed by the function F . Both are interrelated. The situation is similar to one in the group theory of special functions [3]. However, the difference is essential: in the representative dynamics the functional aspects are not derived from the structural ones and have an independent origin.

2. Dynamical inverse problem of representation theory for controlled systems.

In the article [1] dynamical inverse problem of representation theory was considered (see also the review [2] on the general ideology of inverse problems of representation theory). Below this concept will be adapted for the controlled systems. The representative dynamics will play a crucial role.

Definition 2. Let

$$(2) \quad \dot{x} = \varphi(x, u),$$

be the controlled system, where x is the time-dependent m -dimensional complex vector and u is the control parameter. *Dynamical inverse problem of representation theory for the controlled system* (2) is to construct a representative dynamics

$$\dot{\mathbf{X}} = F(\mathbf{X}, a)$$

and the function

$$a = a(u, x) \quad \text{such that} \quad \varphi(x, u) = f(x, a(u, x)),$$

where the operator function F is defined by the Weyl (symmetric) symbol f as a function of m non-commuting variables X_1, \dots, X_m [5:App.1].

Remark 3. If the controls are absent and the pair $a(t) = (\mathfrak{A}(t), \mathbf{e}(t))$ is time-independent, Def.2 is reduced to the definition of the dynamical inverse problem of representation theory of the article [1].

Remark 4. One may consider dynamical inverse problem of representation theory for games, the interactively controlled systems and interactive games (introduced by the author in [4]).

Remark 5. One is able to interpret the correspondence

$$\text{controlled system} \longrightarrow \text{representative dynamics}$$

as a quantization of the first. Such interpretation is very important for the second quantization of intention fields in the interactive games [4].

Remark 6. If the function φ contained some constants $c_\alpha \in \mathbb{C}$ then one may interpret them as time-independent variables and include the matrices $C_\alpha \in \text{Mat}_n(\mathbb{C})$ instead of them in the operator function F (compare with the quantization of constants [2]).

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